Lösungen: Flächenprobleme I

(1) Unter der Oberfläche

$$\int_{-2}^{2} \left(\frac{1}{2}x^2 - 2\right)dx = \left[\frac{1}{6}x^3 - 2x\right]_{-2}^{2}$$

$$= \left(\frac{8}{6} - 4\right) - \left(-\frac{8}{6} + 4\right)$$

$$= \frac{4}{3} - 4 + \frac{4}{3} - 4 = \frac{8}{3} - 8 = -\frac{16}{3}$$

$$\underline{\underline{A = \frac{16}{3}FE}} \approx 5,33FE$$

(2) Die Wiese zwischen 2 Wegen

$$o(x) = 2 - x$$

$$u(x) = x^2$$

$$\int_{-2}^{1} 2 - xdx = \left[2x - \frac{1}{2}x^2\right]_{-2}^{1}$$

$$= \left(2 - \frac{1}{2}\right) - (-4 - 2)$$

$$= 7,5$$

$$\int_{-2}^{1} x^2dx = \left[\frac{1}{3}x^3\right]_{-2}^{1} \quad = \left(\frac{1}{3}\right) - \left(-\frac{8}{3}\right) = \frac{9}{3} = 3$$

$$A = 7,5 - 3 = 4,5$$

$$\underline{\underline{A = \frac{9}{2}FE}}$$

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(3) Berg und Tal

Idee für die Funktion: Regression (4 Punkte)

$$f(x) = x^3 - 2x^2 - x + 2$$

$$\int_{-1}^{1} (x^3 - 2x^2 - x + 2)dx = \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_{-1}^{1}$$

$$= \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right) - \left(\frac{1}{4} + \frac{2}{3} - \frac{1}{2} - 2\right)$$

$$= -\frac{4}{3} + 4 = \frac{8}{3}$$

$$\int_{1}^{2} (x^3 - 2x^2 - x + 2)dx = \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_{1}^{2}$$

$$= \left(4 - \frac{16}{3} - 2 + 4\right) - \left(\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right)$$

$$= \frac{2}{3} - \frac{13}{12} = -\frac{5}{12}$$

$$A = \frac{8}{3} + \frac{5}{12} = \frac{37}{12}$$

$$A \approx \underline{\underline{3,083 FE}}$$

(4) Handtuch

Funktionen:

$$f(x) = \frac{1}{x}$$

$$g(x) = x + \frac{3}{2}$$

$$h(x) = x - \frac{3}{2}$$

Fläche:

$$A_1 = \frac{2 \cdot 2}{2} = 2$$

$$A_2 = \int_{\frac{1}{2}}^{2} \frac{1}{x}dx = [\ln|x|]_{\frac{1}{2}}^{2} = \ln 2 - \ln\frac{1}{2} = \ln 4$$

$$A_3 = \frac{\frac{1}{2} \cdot \frac{1}{2}}{2} = \frac{1}{8}$$

$$A = 2 \cdot \left(2 + \ln 4 - \frac{1}{8}\right)$$

$$A \approx \underline{\underline{6,523 FE}}$$